# A Bayesian Emulator Methodology to Support Evidence-Based Building Energy Model Parameterisation and Uncertainty Analysis

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Mohammad Royapoora, Dario Domingob, Hailiang Dub, Aaron Boranianc, Sara Walkera, Michael Goldsteinb

[a] School of Engineering, Newcastle University, Newcastle upon Tyne, NE1 7RU

[b] Department of Mathematical Sciences, Durham University, Durham, DH1 3LE

[c] Big Ladder Software LLC, Denver, CO 80202

**How many CV(RMSE) and MBE are in each energy subcat that qualify as calibrated model. Could it be that we chose a smaller parameter space that the emulator did not have enough information to be able to completely define the uncertainty space. Run and emulate over a bigger parameter space:**

**However, as the number of spaces increase the**

**Cut out the parameters that are not efficient in enabling expensive models to history match successfully,**

# **Abstract**

This work uses a systematic, evidence-based approach to parameterise a building energy model to be calibrated against monthly gas and electricity data and hourly temperatures. The emphasis is to avoid tuning the model is an ad-hoc manner and retain input parameters within the bounds of actual energy and temperature data, building thermophysical information and observed occupant activities. Expert judgement and product specifications are used to introduce uncertainty bands to train a Bayesian emulator in two annual waves of simulation each involving 1000 runs.

The results show:

Having put the deterministic occupancy and operation of the building as closely as possible, and having used local weather data, the probable variation in fabric properties and plant efficiencies could only bring 7 out of 12 months into a calibration band of +\_15% (CV(RMSE)), with the rest of uncertainties having to be left to micro-climate, weather, stochastic occupant behaviour, ???

# **Introduction**

Stating uncertainty bands and integrating them in building performance simulation results is essential for producing high-quality results that acknowledge modelling limitations. Both model uncertainty and performance gap between model predictions and actual operational data remain areas of active building research. A recent article reported that a large proportion of building modelling community lacks essential knowledge on what the most fundamental parameter inputs for buildings are and how these impact model predictions [1]. This gap has major consequences, as retrofit strategies are mostly derived in consultation with building energy models and as such techno-economic benefits of proposed solutions can be misleading. Similarly, future buildings are expected to be more responsive to other civic activities (i.e. power generation and storage, transport, etc.)[2]. This can only be assessed by modelling, simulation and near-real-time analytics of a cluster of buildings at district and potentially city level which in turn requires generating accurate building energy data.

In order support decisions at private and governmental level, it is crucial that the model being used has been well calibrated. Calibration is the process though which values of the model inputs are chosen, so that model outputs replicate observed data within prescribed tolerance. In the context of building energy models, ASHRAE guidelines [28] are often followed to assess whether a model has been successfully calibrated against observed data [18,42,etc]. The procedure consists in computing the following discrepancy measures between a sequence of simulated model outputs , and the available observations , :

(1)

. (2)

The model is then considered calibrated if the relevant one of the following conditions is met:

1. -10% ≤ MBE ≤ 10% and CV(RMSE) ≤ 30% for hourly data
2. -5% ≤ MBE ≤ 5% and CV(RMSE) ≤ 15% for monthly data.

While the above criteria are easy to check, their use to assess model calibration presents some limitations. We note for example that the criteria do not depend on the level of accuracy to which measurements are available, nor they depend on the level of discrepancy between the real-world and the simulated dynamics of the process under investigation. Such discrepancy will be present due to inherent modelling limitations and assumptions and, as we argue, it could and should be accounted for when model outputs and observations are compared.

Moreover, the computation of MBE and CV(RMSE) can only be carried out after the model has been run at a particular input, and the corresponding output (or sequence thereof) has been observed. This is usually feasible for a limited number of inputs only, due to the time and computational resources needed to perform each run. Albeit unavailable, model outputs may however be predicted through appropriate statistical models, with an associated level of uncertainty. If that is the case, there is no immediate way to include information on the prediction uncertainties into formulas (1) and (2).

Finally, notice that formulas (1) and (2) provide a measure of how average bias and average magnitude of the sequence compare to the average measurement. This is meaningful for positive quantities, such as energy. The dimensionless result is, of course, independent of the unit in which simulated and measured values are expressed in those formulas. The applicability of the formulas however wanes for other physical quantities for which model outputs and observations may be available, such as temperature. In such a case, different units yield different results. The International System of Unit (Kelvin) should be used to ensure that all quantities are positive, but criteria a) and b) would then be fulfilled even for inputs for which simulated and observed temperatures are far from each other, owing to the large value of the denominator in expressions (1) and (2).

* Add some sentences on the use of Bayesian calibration (in general and in energy literature) to address some of the above challenges.

The aim of this study is three-fold. First, to review sources of uncertainties often overlooked in model calibration and to provide a statistical framework to account for them in a unified manner. Second, to design a procedure which iteratively refocuses on subregions of the model input space where a match between model outputs and observed data can be found, in light of the above uncertainty framework. Third, to illustrate the procedure on a building-energy case study, where uncertainties are quantified and observed monthly consumptions are used to calibrate the model.

In order to assess how well a given model input replicates observed data we make use of the so-called implausibility measures. They have already been employed in a variety of disciplines to reconcile model outputs and data, for example in the context of ice sheet shapes [47], galaxy formation [44], hydrocarbon reservoirs [46], gene evaluation [45]. Similarly to the expression in equation (1), implausibility measures quantify the distance between model outputs and observations. However, they do so in light of the magnitude of uncertainties affecting each of the two sources, and potentially others.

Finally, we note that the methodology we propose relies on the use of a statistical surrogate of the original model, often called an emulator. Once trained and validated, an emulator provides instantaneous predictions of the model outputs at inputs at which the model has not been evaluated. In the context of calibration, this allows a thorough exploration of the model input space. Moreover, one of the key feature of an emulator, alongside its speed, is that it provides an estimation of the uncertainty associated with each prediction it makes. The framework we lay out easily allows to account for this additional source of uncertainty in the calibration process.

1. **UNCERTAINTY QUANTIFICATION OF COMPUTER MODELS**

**2.1 Calibration of Computer Models under Uncertainty**

Calibration of computer models is a crucial topic not only in the area of energy models, but in a variety of other scientific disciplines. It consists in identifying values of the inputs of a computer model whose corresponding outputs match observed data. Mathematically, the model can be seen as a function *f*, taking as input *x* the model parameters of interest, and returning *f(x)* as output. Hence, denoting by z the real-world observations of the process modelled by the simulator, the calibration task can be stated as finding the inputs x so that f(x) is a “close enough” match to z.

Proximity between the two quantities should be assessed upon consideration of all sources of uncertainty that affect the system. While an exhaustive list of such sources is problem-dependent and challenging to specify in its entirety [refs to some of Michael’s papers], two sources of uncertainty usually play a prominent role in the calibration of building energy models and many others.

1. *Model Discrepancy.* Modelling assumptions and numerical approximations, albeit unavoidable, make any model an imperfect representation of reality. The simulated output f(x) will be different from the (average) value that would be observed in the real world under the conditions specified in x. We refer to this difference as model discrepancy, which we discuss in more detail for energy models in Section 2.2.
2. *Measurement Error*. Energy consumption observations, as much as any observation, suffer from measurement errors. Quantification of their magnitude can often be found in the specifications of the measuring device used.

Accounting for the above sources of uncertainty is crucial when assessing whether a model has been successfully calibrated against observed data. To accomplish the task, in Section 3.3 we discuss the use of the so-called implausibility measures. These allow the researcher to include various sources of recognised uncertainty when model outputs and observations are compared.

In addition to the presence of model inaccuracies and measurement errors, one feature that makes calibration challenging is the feasibility of extensively exploring the model input space. In fact, a uniform covering of the space requires a number of points that grows exponentially with the input space dimension. Hence, running the corresponding number of simulations proves prohibitive even in fairly low dimensions.

To meet the computational challenges involved, a new area of Bayesian statistics has been developed over the last few decades, the one of emulation. An emulator is a statistical surrogate of the simulator, providing predictions of the simulator’s output at inputs *x* which the simulator has never been evaluated at. The main advantage of using the emulator over the simulator is that the former, once built and validated, is instantaneous to run and reliably quantifies the uncertainty associated with the prediction. All it is needed for its construction is a (small) set of inputs and associated simulator’s outputs, on the basis of which the statistical model is created. Emulator predictions can then be computed in milliseconds on a personal laptop, regardless of the complexity and computational expensiveness of the original model.

The methodology we present in this study relies on the use of emulators to perform a thorough exploration of the model parameter space. Alongside the use of implausibility measures, this allows to exclude regions of the input space where a match with the observed data can be confidently ruled out, and to identify the region where, all uncertainties considered, a match is possible.

**2.2 Model Discrepancy in Building Energy Systems**

Multiple factors drive energy consumption in a building, such as quality of wall insulation, external weather, occupants’ lifestyle, etc. While current models such as Energy+ attempt to reproduce the physics of \*\* with high fidelity, no model can include an exact representation of all the features affecting environmental properties (*e.g.* internal temperature) and energy consumption in a building. Simplifications are necessary to keep the model complexity to reasonable levels. Example of factors which are commonly either neglected or included via simpler dynamics are as follows:

1. The dynamic nature of fabric thermo-physical properties: most models consider the U-value of masonry walls as constant, although its value changes as a function of wall moisture content. *(Hygroscopic material can indeed exhibit wide thermophysical variance at different moisture and temperature conditions.)*
2. The occupant’s behaviour and interaction with the building (window opening, light and small power usage): their stochastic nature is rarely accounted for in energy models.
3. Uncertainties and variations in plant operational characteristics: these are extensively simplified in energy models.
4. Exact zone air exchange figures and fabric infiltration values: they are difficult to determine, and fabric infiltration values can only be estimated after performing a building pressure test that is logistically difficult and costly, particularly for larger occupied buildings.
5. The available weather files: that impose large uncertainties in particular with solar irradiance data that quite often is partially or fully modelled (as opposed to measured). Micro-climatic variations are widely ignored and understudied. An example could be the difference in conventional airport weather station data used to model urban settings. Airports are exposed terrains often with proximity to water to facilitate emergency aircraft landing. Annual weather files compiled in these locations would therefore report higher wind velocities and miss the heat-island effect that is increasingly separating urban and open country micro-climates.

None of the above assumptions and approximations undermines the validity of the model. The latter remains a key tool for the researcher to gain insight on the real-world phenomenon under study, such as energy consumption in a building. However, we advocate here that it is key to acknowledge the discrepancy between model and reality when outputs from the model are compared to real-world observations, *e.g.* in a calibration task.

1. **METHODOLOGY**

The flowchart in Figure 1 illustrates the methodology we propose. This aims to sequentially rule out regions of the input space where a match between model outputs and observations can be confidently rejected given the involved uncertainties. We mainly make use of two tools to accomplish the task:

1. Emulators: to instantaneously predict the model output at inputs where the model has not been run, and quantify the prediction error;
2. Implausibility measures: to statistically quantify the “distance” between model outputs and observations, in light of different sources of uncertainties.

The first two steps of the methodology (blue in Figure 1) consist in quantifying the uncertainties in the model and in the observed data (energy consumption for the sake of this work, but potentially any observable quantity), and in training an emulator of the model in question. The two steps are independent of each other, and are discussed in Section 3.1 and 3.2 respectively. The following steps (yellow in flowchart) require to evaluate the implausibility measure at a very large number of inputs x over the space where matches are being sought, so that regions with highly implausible matches can be ruled out. At this point, the researcher may wish to repeat emulator training and implausibility evaluation on the Not-Ruled-Out-Yet (NROY) part of the space: the simulator often displays simpler, more linear behaviour locally, hence a new emulator over the smaller region can be more precise and allow the researcher to exclude further regions (red). Sections 3.3 details the definition of implausibility measures, while their use for the above tasks are discussed in Section 3.4.

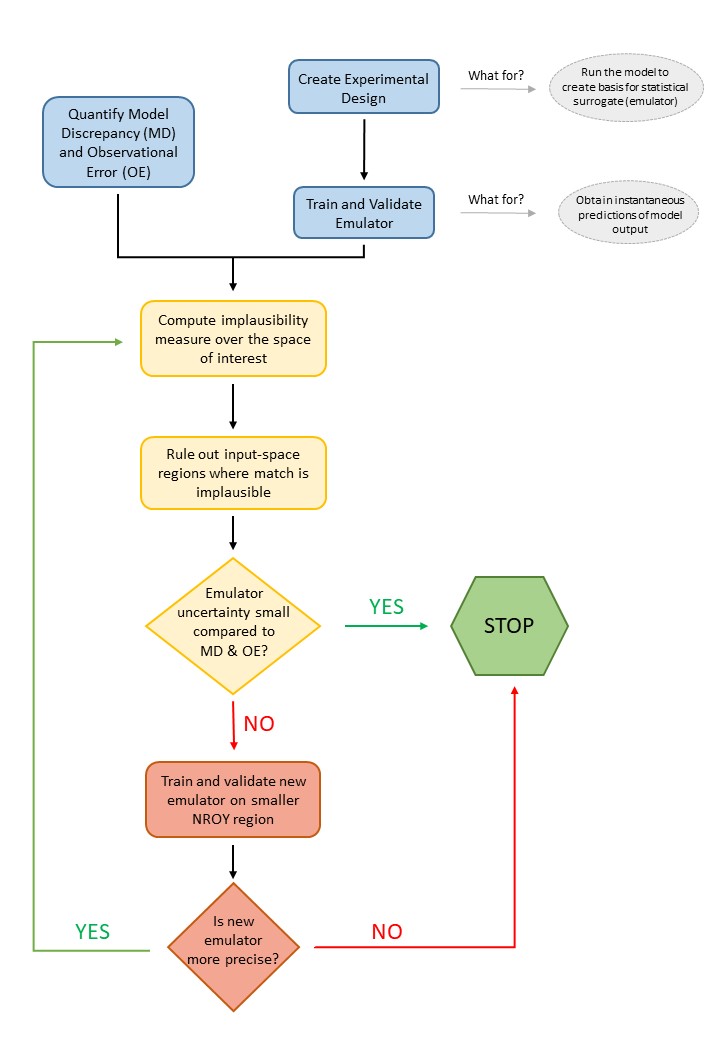


Figure 1: Flow chart illustrating the methodology proposed in this study.

* 1. **Quantifying Model Discrepancy and Measurement Errors**

The concepts of model discrepancy (MD) and observational error (OE) are not unique to the context of energy model calibration. They are in fact relevant to any uncertainty analysis linking computer models and measurements. For the sake of illustration, in the following the reader may think of the computer model *f* as predicting energy consumption in a building, as a function of system properties *x* such as the ones in Table \*\*. The validity of the structure we present here is however independent of the specific problem, and has in fact been successfully applied to a wide range of contexts [44-47].

Following [43], we assume that an appropriate choice of model inputs exists, x\*, that accurately represents the values of the system’s properties. The real-world value *y* of the modelled quantity and the simulator output *f(x\*)* at the above choice of inputs are then linked via the formula:

where the model-discrepancy term accounts for the difference between real and simulated process.

The (unobservable) real-system value *y* is usually estimated through measurements *z* of the process of interest. We link these two quantities via the relationship:

where the term accounts for the observational error in the measurements.

The additive formulation in equations (1) and (2) is a simple but efficient way to model MD and OE, which also makes statistical inference tractable. Information about the quantities and should in fact be sought in statistical form, rather than be quantified as a single number.

Manufacturer guidelines are usually available to estimate the magnitude of OE (*e.g*., up to 5% of the measured value). Estimates of MD magnitude may be slightly more challenging to obtain. However, the modeller’s knowledge of the assumptions and approximations used within the simulator, alongside literature research, usually provide guidance on the uncertainty effects of different assumptions, and can therefore lead to an overall estimate of the discrepancy between the model and reality at the “best” (unknown) inputs *x\**. In the case study discussed in this paper (Section 3) we consider both 10% and 20% MD, and discuss how the choice affects calibration results. For a more detailed treatment of MD, and the further distinction between internal and external MD, the reader is referred to [43].

* 1. **Bayes Linear Emulators as Fast Model Surrogates**

\* Say what an emulator is and what is needed to train one

\* Illustrate main idea, but refer to literature for details. Do stress however that a number of choices should be made (the level of detail here is to be seen)

\* Possibly provide Dario’s Github link to R implementation of a function performing emulation.

* 1. **Implausibility Measure**

\* Refer to notation of Sec 2.1, hence stress we want to compare real-world phenomenon y with the values we would observe under conditions x (f(x)+ model discrepancy).

\* Can only compare f(x) and z. Moreover, f(x) unknown for most x. Replace f(x) by emulator, hence provide formula for IM.

* 1. **History Matching (calibration)**

\* Need to decide on a threshold for the IM, above which inputs x are deemed implausible and discarded.

\* Classical threshold can be 3. However, left to the judgement of the researcher. May use higher values for example if multiple observations are available hence more than one condition is being imposed.

\* Usually proceed in waves. At each wave, fit a new emulator based on training within the NROY region only. This is to get a more precise emulator in that region. Stop when emulator uncertainties do not improve within the NROY region found, or when emulator uncertainty is anyway negligible compared to model discrepancy and measurement error.

1. **CASE STUDY**

**4.1 Building**

A detached two-storey masonry construction built in 1994 was selected as the case-study building. Two occupants are the only residents of the dwelling and were asked to archive their gas cooker and shower usage each day across an annual cycle. Given a very predictable pattern of occupancy (both occupants had 8am-5pm working commitments), it was possible to limit the stochastic nature of occupant activity as far as practically manageable and use deterministic schedules to represent occupant activity. The building (with a gross area of 168.66m2 and 19.73m2 of unheated space) is in a built-up urban surrounding, is only partly shaded (on its west elevation) by another adjacent property (which was considered within the modelling work). Across the monitoring year (2016) the property had an observed annual gas (15,381 kWh) and electricity (2,991 kWh) consumptions that respectively reflect high and medium UK typical domestic consumption values [23]. Occupants only utilised shower facilities which at a measured flow rate of 4.37 l/min and recorded average eight 20-minute showers per week correspond to an average of 50 l/person/day. These recoded values are below UK average domestic hot water usage (reported as 142 l/person/day [24] and 122 l/person/day [25]), but primarily reflect their heavy use of gym washing facilities. Gas cookers (containing 3kW and 5kW hubs) were used 4 times a week for 1 hr per cooking session.

**4.2 Energy and temperature data collection**

A proprietary set of environmental and energy sensors were deployed to compile electricity and zone temperatures (Fig. 1). To reduce measurement uncertainty, each of the two target zones was equipped with two separate air temperature sensors at 1.3m above floor level and set to log data at 30s intervals to achieve a moderated average. Therefore, space temperature was recorded by 4 sensors (two positioned in the south facing master bedroom and two in north facing kitchen). Whereas electricity and gas data required no imputation, overall kitchen and master bedroom temperature sensors had total annual losses of 5.7% and 2.7% that required imputation. Each missing hourly temperature cell was imputed by the average of the previous and successive available cells. Gas consumption data was manually recorded at monthly basis using mains gas meter. Electricity was logged at 10s intervals using two clip-on current sensors on the incoming live cable (to reduce measurement uncertainty) and the two sets of similar readings were averaged and aggregated to form the measured power usage.



Figure 1 LSH: power monitor used to characterise household appliances, RHS: AC current sensor, monitoring transmitters and temperature sensors deployed in case-study building.

In order to parameterise the energy model more accurately, a plugin power monitor was also used to characterise instantaneous and time-average consumption of the main electrical devices (TV, washing machine, ICT) in the property.

**4.3 Input parameter range selection**

By consulting manufacturers specification and the house builder’s literature, a detailed set of parameter inputs were compiled and where the greatest quantifiable uncertainty existed, lower and upper bands were imposed on the input value used. These bands were so far as possible derived from scientific literature and used to dictate the size of associated variations explored in batch-runs (Table 2). Ground floors are less prone to variations in internal and external air velocities that act on walls and roofs more robustly, and lead to dynamic heat transfer values that fail to be captured by standardised calculation methods. Therefore, a smaller floor uncertainty margin was derived from literature and imposed on floor thermal resistance (Tables 2 and 3).

The property’s glazing was updated in 2009 and manufacture’s literature set the G and U-value of the fenestration to 0.691 and 1.788 W/m2K respectively, with respective error bands of ±5% and ±2%. The compound upper and lower limit of these two values altered the gas consumption of the calibrated model by ±2.05 kWh (± 0.013%). Given its negligible nature, the error bands of the glazing were discounted in batch simulations. The compound effect of all other uncertainty bands created a lower boundary of 8,842 kWh and an upper boundary of 26,452 kWh with respect to an observed gas consumption of 15,381 kWh (i.e. -42.5% to +72%). Given that even a 5% increases in fabric U-value was reported to raise energy consumption of family homes by 0.3-2.5% [26], a uniformly distributed uncertainty band is imposed on elemental U-Values to reflect similar magnitude of variations reported in literature, as outlined in Table 2.

Actual building infiltration rates are difficult to arrive at and require convoluted air permeability tests. Table 4.16 of CIBSE guide A [6] outlines a range of 0.25 to 0.95 air change per hour (ACH) for various 2-storey buildings below 500m2 with a value of 0.5 ACH describing typical constructions similar to the case-study building. Therefore 0.5 ACH informed the calibrated base model with 0.25-0.95 ACH representing the range of possibilities that batch simulations in the Bayesian emulator explored (Tables 2-3). Local weather files compiled by a weather station approximately 3 miles away from the site was used to support the model development [27].

Table 2 Parameter inputs for energy model development of the case-study building

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Description** | Uncertainty range |
| Heating | Natural gas boiler serving a radiator central heating system |  |
| Heating setpoint (setback) | 19°C (16°C) | 17.5°C-20.5°C |
| Heating schedule | 02:00-11:00 + 16:00-24:00 |  |
| Ventilation | Natural ventilation (mechanical extract to family bathroom and en suite) |  |
| Ventilation rate | Highly stochastic, controlled by occupants via openable windows |  |
| Gas boiler seasonal efficiency | 65% (15 years old non-condensing gas-fired system boiler – 77°C/55°C F+R) | 60% - 75% |
| DHW consumption | 0.59 litre/m2/day |  |
| Cooling setpoint (setback) | Uncontrolled |  |
| Nominal lighting power density | 1.4 W/m2 (manually controlled) to achieve 200 lux |  |
| Occupants | 2 people in total |  |
| Internal gains[a] | 6 W/m2 |  |
| Gross (conditioned) area | 168.66m2 (148.93m2) |  |
| Observed annual gas (electricity) consumption (2016) | 15,381 kWh (2,991 kWh) |  |
| **Fabric properties:** |  |  |
| Glazing (with low emissivity coating) | 1.788 W/m2K (3mm self-cleaning pane, 20mm Argon filled cavity, 3mm low emissivity pane) | |
| Glazing G Value (solar transmittance) | 0.691 |  |
| External walls [b] ( W/m2K) | 0.544 | ± 15% |
| Roof [c] (W/m2K) | 0.213 | ± 15% |
| Floor [d] ( W/m2K) | 0.335 | ± 5% |
| Infiltration (ac/h) [e] | 0.5 | 0.25 - 0.95 |
| [a] Electricity (ICT and appliances): 3 W/m2; Gas (catering): 3.3 W/m2 | | |
| [b] 100mm brickwork, 50mm Stone wool insulation, 100mm blockwork, 10mm plasterboards | | |
| [c] 25mm Clay tile roofing, loft space, 180mm glass fibre quilt insulation, 10mm plasterboards | | |
| [d] 100mm cast concrete, 7mm screed, 4mm high gauge polythene DPM, 5 mm foil-backed underlay, 15mm solid wood flooring | | |
| [e] Empirical values derived from table 4.16 (CIBSE Guide A) for a two-storey property on normally exposed site | | |

1. **RESULTS**

This section illustrates an application of the methodology we propose, taking as case study the one presented in Section 4. We first build emulators of some of the outputs of the building energy model under consideration, using these to carry out a thorough exploration of the model’s input space in a reduced amount of time. Hence, in a statistical framework accounting for various sources of uncertainties, we use the emulator predictions to identify subsets of the input space which yield matches with the observed data.

* 1. **Simulation Runs**

Table 3 shows the eight variables varied in the model. For each of these, the range of values within which a match is sought is reported (motivations for the choice of ranges discussed in Section 4.3).

In order to build emulators of the monthly energy consumptions, the outputs of a small number of model runs are needed. An informal rule of thumb suggests to use about 10 runs for input dimension [48]. This is only an indication: a lower (higher) number of runs may prove sufficient (necessary) to build an accurate emulator, according to the complexity of the model’s response to its inputs. In our case the simulator was reasonably fast to evaluate (*specify estimate of time*), and a total of 1,000 simulations were run - substantially more than what typically needed for eight inputs.

The 1,000 points identifying inputs for the same number of runs were chosen through Latin Hypercube Sampling (LHS). LHS is a near-random sampling technique, allowing to sample points in a hypercube that fill the space while not being too close to each other [49]. For this reason it is commonly used to identify a set of runs whose outputs are used to create a statistical surrogate of the simulator’s behaviour throughout the space, see *e.g.* [44], [50], [51]. In R, LHS can be easily performed through the “lhs” package.

Figure of outputs Outputs

* Show histograms of gas outputs compared to measured data (histograms ready). State that summer months will be neglected for the analysis, leaving 9 months to match.
* Electricity runs show almost no variation, and when they do simulated values are far from observed target. Neglect them.
  1. **Emulation of Monthly Gas Consumption**
* Provide details on split of runs (training, evaluation, test), choice for regression factors and active inputs for emulator. Probably mention validation (?) but no space to show plot here. Appendix/supporting material?
* Comment on remarkable emulator precision, eg through empirical confidence intervals of predicted standard deviation on test inputs (table is ready).
  1. **Observational Error and Model Discrepancy**
* Observational error set to 5% (Mohammed may give some justification).
* For illustrative purposes, results shown with 10% and 20% MD.
  1. **Calibration (history matching)**
* Implausibility measures I\_M computed at each month M, hence overall implausibility I computed as maximum. Input considered non-implausible if I<4.
* Posterior distribution of inputs reveals in particular 3 significant variables. Show min-implausibility and optical-depth plots for these 3, in the two cases 10% and 20% MD.
* Discuss features such as: contribution of single months in cutting out space (September has a lot, while eg Jan/Feb almost interchangeable).

1. **DISCUSSION AND CONCLUSIONS**

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Table 1: Sequence of uncertainty analysis on building energy models

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Field | Model inputs | Input range | Distribution of uncertainty | Reported variation in model output |
| Building energy model | Occupant behaviour | Presence, density, heat gain | Binary and non-binary | No reports found | 30% [31]  4-26% [32] |
| Building envelope thermal properties | U-values, thickness, surface and moisture properties | Range using Mean and SD [33] | Even [22]  Normal [33] | 42%[34] |
| Weather conditions | Wind speed, direction and pressure coefficients, solar irradiance, air humidity and temperatures | Cold, Med, Hot [33] | Normal [35]  Bivariate Normal [36]  Discrete Distribution [33] | -4% to 6.1% [37] |
| Site micro-environment | Wind-pressure coefficient, ground albedo | Range using Mean and SD [33] | Normal [33] | Not reported |
| HVAC | Values assumed for CoP, SEER and η | Best practice, typical | Normal Distribution [22]  Gamma Distribution [38] | -15.3% -70.3% [37] |
| Internal Gains |  | Low, Med, High [33] | Uniform discrete [33] |  |
| Operational regime | Controls and Scheduling of all HVAC, lighting and plug-in items | Good, average and poor practice [37] | Uniform Discrete [37] | -28.7%-79.2% [37] |
| Observational data | Gas [1] | - | - | Normal [22] | n/a |
| Electricity [1] | - | - | Normal [22] | n/a |
| Temp (Kitchen) | - | - | Normal [22] | n/a |
| Temp (master) | - | - | Normal [22] | n/a |
| Notes  [1] The gas and electricity meters’ accuracy were expected to comply with SI 684 (1983) and IEC 62053 respectively that allow +2.5% or −3.5% of compound instantaneous deviations. | | | | | |

Table 2 Parameter inputs for energy model development of the case-study building

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Description** | Uncertainty range |
| Heating | Natural gas boiler serving a radiator central heating system |  |
| Heating setpoint (setback) | 19°C (16°C) | 17.5°C-20.5°C |
| Heating schedule | 02:00-11:00 + 16:00-24:00 |  |
| Ventilation | Natural ventilation (mechanical extract to family bathroom and en suite) |  |
| Ventilation rate | Highly stochastic, controlled by occupants via openable windows |  |
| Gas boiler seasonal efficiency | 65% (15 years old non-condensing gas-fired system boiler – 77°C/55°C F+R) | 60% - 75% |
| DHW consumption | 0.59 litre/m2/day |  |
| Cooling setpoint (setback) | Uncontrolled |  |
| Nominal lighting power density | 1.4 W/m2 (manually controlled) to achieve 200 lux |  |
| Occupants | 2 people in total |  |
| Internal gains[a] | 6 W/m2 |  |
| Gross (conditioned) area | 168.66m2 (148.93m2) |  |
| Observed annual gas (electricity) consumption (2016) | 15,381 kWh (2,991 kWh) |  |
| **Fabric properties:** |  |  |
| Glazing (with low emissivity coating) | 1.788 W/m2K (3mm self-cleaning pane, 20mm Argon filled cavity, 3mm low emissivity pane) | |
| Glazing G Value (solar transmittance) | 0.691 |  |
| External walls [b] ( W/m2K) | 0.544 | ± 15% |
| Roof [c] (W/m2K) | 0.213 | ± 15% |
| Floor [d] ( W/m2K) | 0.335 | ± 5% |
| Infiltration (ac/h) [e] | 0.5 | 0.25 - 0.95 |
| [a] Electricity (ICT and appliances): 3 W/m2; Gas (catering): 3.3 W/m2 | | |
| [b] 100mm brickwork, 50mm Stone wool insulation, 100mm blockwork, 10mm plasterboards | | |
| [c] 25mm Clay tile roofing, loft space, 180mm glass fibre quilt insulation, 10mm plasterboards | | |
| [d] 100mm cast concrete, 7mm screed, 4mm high gauge polythene DPM, 5 mm foil-backed underlay, 15mm solid wood flooring | | |
| [e] Empirical values derived from table 4.16 (CIBSE Guide A) for a two-storey property on normally exposed site | | |

Table 3 Input parameter variations for Bayesian emulator development

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| Description | Heating setpoint [17.5°C-20.5°C] | Boiler seasonal efficiency | External wall U-value | Roof U-Value | Floor U-value | Infiltration rate (ach) | DHW consumption  (L/day/person) | Cooking |
| Base model input (1st wave) | 19°C | 65% | 0.544 | 0.213 | 0.337 | 0.5 | 300 (1st wave)  120 (2nd wave) | 3% of total domestic energy use |
| Base model input (2nd wave) | 17.5°C | 65% | 0.544 | 0.213 | 0.337 | 0.5 | 120 |  |
| Range of variation | ± 1.5°C | 60% – 75% | ± 15% | ± 15% | ± 5% | 0.25 to 0.95 | 70-250 L/day | 1.05% - 6.3% |
| Rational | [a] | [b] | [c] | [c] | [d] | [e] | [f] | [g] |
| Uncertainty quantification | Forward | Forward | Forward | Forward | Forward | Forward | Inverse | Inverse |
| Element varied in E+ batch simulations | Heating setpoint | Boiler seasonal efficiency | Wall cavity Insulation thickness [ 40mm-63mm] | Insulation thickness [150mm -210mm] | Insulation thickness [45mm- 55mm] | [e] |  |  |
| [a] Manufacturer’s room thermostat resolution reported at ± 0.5°C with an additional ± 1°C allowed for time-dependent drift degradation.  [b] Boiler insulation, heat exchanger and working fluid degradation, limescale and total dissolved solids leading to an accumulated min and Max performance degradation of 4% to 23% [39, 40]. These levels of degradation were imposed on boiler manufacturer’s quoted efficiency of 78%  [c] Although most literature report in-situ wall and roof measurements to be better than elemental method calculation suggestions [14, 16, 17], an equally distributed ± 15% imposed to first cater for all eventualities and enable the uncertainty emulator to assess the entire Latin hypercube space (including worst scenario range).  [d] as per [c] although the magnitude of variations reported for floors were smaller than those of walls/roofs [17] and non-suspended ground floors with no air cavities have much greater thermal unity [41] so a tighter band of ± 5% was imposed to reflect literature findings.  [e] As outlined in the last paragraph of sections 2 and 4.  [f] From field measurements of DHW consumption in the UK [25] where the mean DHW consumption per person in the UK is reported as 122 litres/day ± 18 litres/day (i.e. ±15% variation) leading to mean DHW energy consumption of 16.8 MJ/day ± 2.2 MJ/day (95% statistical confidence). In the 1st wave the model input was a much larger values of >300l/day and 53MJ/day, However the model predictions were calibrated to return close results to the observed energy consumption for the case-study building given its high fossil fuel consumptions.  [g] Cooking has been observed to currently account for an average of 3% of total household energy demand with historical data also indicating a maximum of 6% [30]. This observed data informs the average and maximum cooking demand with 1% also selected by the authors to represent a probable lower boundary. | | | | | | | | |